Low Tucker Rank Tensor Completion Using a Symmetric BCD Method

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2021.10.10

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Definition (Tensor)

A tensor is a multidimensional array. More formally, an N-way or Nth-order tensor is an element of the tensor product of N vector spaces, each of which has its own coordinate system.

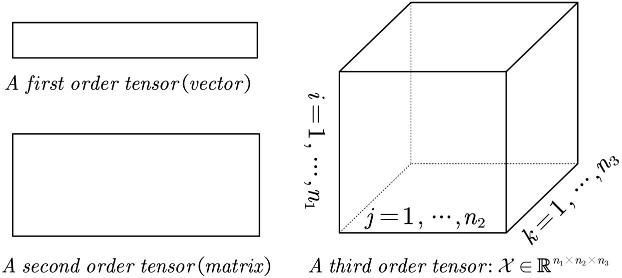
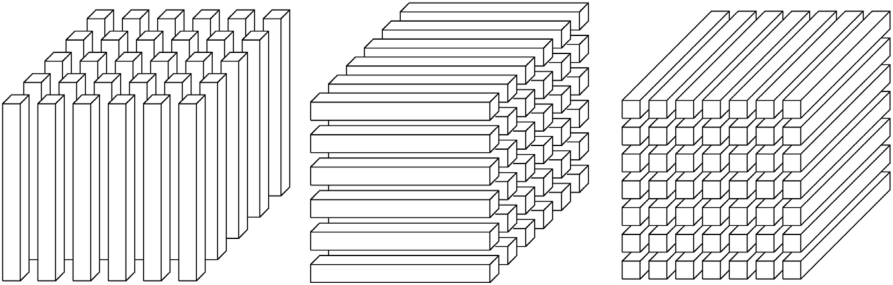


Figure: tensor.

Definition (Fibers)

Fibers are the higher-order analogue of matrix rows and columns. A fiber is defined by fixing every index but one.

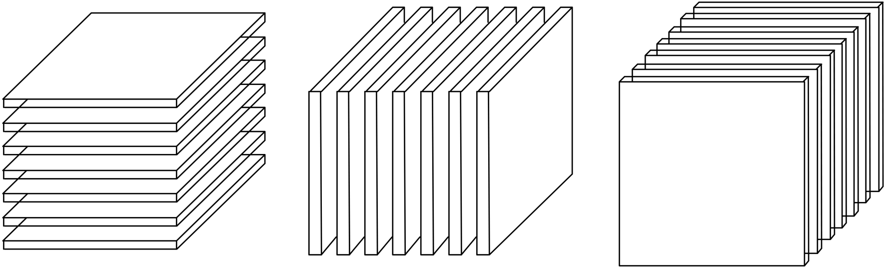


(a) Mode-1 (col) fibers: $x_{:jk}$ *(b)* Mode-2 (row) fibers: $x_{i:k}$ *(c)* Mode-3 (tube) fibers: $x_{ij:}$

Figure: Fibers of a 3rd-order tensor.

Definition (Slices)

Slices are two-dimensional sections of a tensor, defined by fixing all but two indices.



(a) Horizontal slices: $X_{i::}$ *(b)* Lateral slices: $X_{:j:}$ *(c)* Frontal slices: $X_{::k}$

Figure: Slices of a 3rd-order tensor.

CANDECOMP/PARAFAC Decomposition

Definition (CP Decomposition)

Suppose that $\mathcal{X} \in \mathbb{R}^{n_1 \times n_2 \times \cdots \times n_m}$. If there exist $a_r^{(i)} \in \mathbb{R}^{n_i}$ for $i = 1, \ldots, m$ such that

$$\mathcal{X} = \sum_{r=1}^{R} a_r^{(1)} \circ a_r^{(2)} \circ \cdots \circ a_r^{(m)}, \quad a_r^{(k)} \in \mathbb{R}^{n_k}. \tag{1.1}$$

The (1.1) is said to be a CP decomposition of $\mathcal{X}$. The smallest R of (1.1) is called CP-rank, denoted by $rank_{CP}(\mathcal{X})$. The CP decomposition with $R = rank_{CP}(\mathcal{X})$ is called a CP rank decomposition.

Remark: It is NP-hard to determine CP rank.

Tucker rank

Definition

For a tensor $\mathcal{X} \in \mathbb{R}^{n_1 \times n_2 \cdots \times n_m}$, let $X_{(i)} \in n_i \times N_i$ for $i \in [m]$. The Tucker rank (or n-rank)of $\mathcal{X}$ is

$$\text{rank}_{tc}(\mathcal{X}) = \left(\text{rank}(X_{(1)}), \cdots, \text{rank}(X_{(m)})\right),$$

where $N_i = n_1 \times \cdots \times n_{i-1} \times n_{i+1} \times \cdots \times n_m$.

Remark: Unfolding a tensor directly will destroy the original multi-way structure of the data, which leads to vital information loss and degraded performance.

Tubal rank

For $\mathcal{A} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$, let $\bar{\mathcal{A}} \in \mathbb{C}^{n_1 \times n_2 \times n_3}$ be the result of Discrete Fourier transformation (DFT) of $\mathcal{A} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$ along the mode-3. Specifically,

$$\bar{\mathcal{A}}(i, j, :) = F\mathcal{A}(i, j, :),$$

Where $F \in \mathbb{C}^{n_3 \times n_3}$ is the Fourier matrix.

Definition

For any tensor $\mathcal{A} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$, let $r^l = rank(\bar{A}^{(l)})$ and $l \in [\mathbf{n_3}]$. Then tubal rank of $\mathcal{A}$ is defined as $rank_t(\mathcal{A}) = \max\{r^1, r^2, \ldots, r^{n_3}\}$.

Remark: Tubal rank is applicable only for third-order tensors.

Low rank tensor completion

Low rank tensor completion:

$$\min_{\mathcal{C}} rank(\mathcal{C}) \qquad \text{s.}t. \quad P_{\Omega}(\mathcal{C}) = P_{\Omega}(\mathcal{M}), \tag{1.2}$$

where $rank(\cdot)$ is a tensor rank and Ω is an index set locating the observed data. P_{Ω} is a linear operator that extracts the entries in Ω and fills the entries not in Ω with zeros, and $\mathcal{M}$ is a given tensor.

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Tucker rank is a vector of matrix ranks, which makes tensor rank minimization problem be a vector optimization. To keep things simple, we use weighted Tucker rank as a tensor rank in low rank minimization problem (1.2), written as

$$\min_{\mathcal{X}} \sum_{i=1}^{m} \operatorname{rank}(X_{(i)}) \quad \text{s.t.} \quad P_\Omega(\mathcal{X}) = P_\Omega(\overline{\mathcal{X}}). \tag{2.1}$$

(1) Romera-Paredes and Pontil proved that the average of the nuclear norm of different unfolding matrices of a tensor is not the convex envelope of the sum of the rank of the unfolding matrices of the tensor;

(2) Unfolding a tensor directly will destroy the original multi-way structure of the data, which leads to vital information loss and degraded performance.

Combining the definition of Tucker decomposition, we consider the following multilinear matrix rank minimization problem

$$\begin{aligned}
\min_{\mathcal{H}, V^{(1)}, \ldots, V^{(m)}} \quad & \sum_{i=1}^{m} \operatorname{rank}\left(V^{(i)}\right) \\
\text{s.t.} \quad & P_\Omega\left(\mathcal{H} \times_1 V^{(1)} \times_2 \cdots \times_m V^{(m)}\right) = P_\Omega(\overline{\mathcal{X}}), \\
& \mathcal{H} \in \mathbb{R}^{n_1 \times n_2 \times \cdots \times n_m}, \; V^{(i)} \in \mathbb{R}^{n_i \times n_i}, \quad i \in [m].
\end{aligned} \tag{2.2}$$

The relationship between (2.1) and (2.2)???

Theorem

Problem (2.1) and problem (2.2) are equivalent. That is, they have the same optimal values.

Since the truncated nuclear norm achieves an accurate and robust approximation to the rank function, we adopt it in problem (2.2) to relax matrix rank function. From the structure of $\mathcal{H}$ and $V^{(i)}$, it is observed that $\mathcal{H}$ and $V^{(i)}$ ($i \in [m]$) are all sparse. Hence, we consider the following relaxed problem

$$\begin{aligned} \min \quad & \sum_{i=1}^{m} \left(\left\| V^{(i)} \right\|_{*,r_i} + \lambda \left\| V^{(i)} \right\|_{l_1} \right) + \lambda \left\| \mathcal{H} \right\|_{l_1} \\ \text{s.t.} \quad & P_\Omega \left(\mathcal{H} \times_1 V^{(1)} \times_2 \cdots \times_m V^{(m)} \right) - P_\Omega \left(\overline{\mathcal{X}} \right) = 0, \\ & \mathcal{H} \in \mathbb{R}^{n_1 \times n_2 \times \cdots \times n_m}, \; V^{(i)} \in \mathbb{R}^{n_i \times n_i}, \quad i \in [m]. \end{aligned} \tag{2.3}$$

To solve such problem, we further introduce surrogate tensor variable $\mathcal{M}$ and matrices $W^{(i)}$ to rewrite the problem as:

$$\begin{array}{ll} \min & \sum_{i=1}^{m} \left(\left\| V^{(i)} \right\|_{*,r_i} + \lambda \left\| W^{(i)} \right\|_{l_1} \right) + \lambda \left\| \mathcal{H} \right\|_{l_1} \\ \text{s.t.} & P_{\Omega}\left(\mathcal{M} - \overline{\mathcal{X}} \right) = 0,\ \mathcal{M} = \mathcal{H} \times_1 V^{(1)} \times_2 \cdots \times_m V^{(m)},\ W^{(i)} = V^{(i)}, \\ & \mathcal{H} \in \mathbb{R}^{n_1 \times n_2 \times \cdots \times n_m},\ V^{(i)} \in \mathbb{R}^{n_i \times n_i}, \quad i \in [m]. \end{array} \tag{2.4}$$

By penalizing the constraint $\mathcal{M} = \mathcal{H} \times_1 V^{(1)} \times_2 \cdots \times_m V^{(m)}$ and $W^{(i)} = V^{(i)}$, we get the following problem

$$\begin{aligned} \min \quad & \sum_{i=1}^{m} \left(\mu \left\| V^{(i)} \right\|_{*,r_i} + \mu\lambda \left\| W^{(i)} \right\|_{l_1} + \frac{1}{2} \left\| W^{(i)} - V^{(i)} \right\|_F^2 \right) + \mu\lambda \left\| \mathcal{H} \right\|_{l_1} \\ & + \ell \left(\mathcal{H}, V^{(1)}, \ldots, V^{(m)}, \mathcal{M} \right) \\ \text{s.t.} \quad & P_\Omega \left(\mathcal{M} - \overline{\mathcal{X}} \right) = 0. \end{aligned} \tag{2.5}$$

Here $\ell \left(\mathcal{H}, V^{(1)}, \ldots, V^{(m)}, \mathcal{M} \right) = \frac{1}{2} \left\| \mathcal{H} \times_1 V^{(1)} \times_2 \cdots \times_m V^{(m)} - \mathcal{M} \right\|_F^2$ and $\mu > 0$ is a penalty parameter. It is well known that an optimal solution of (2.5) approaches an optimal solution of (2.4) as $\mu \to 0$.

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Update $\mathcal{H}$

Core tensor $\mathcal{H}^{(i,k)}$ can be updated by solving the following problem

$$\min_{\mathcal{H}^{(i,k)}} \ \mu\lambda \left\|\mathcal{H}^{(i,k)}\right\|_{l_1} + \ell\left(\mathcal{H}^{(i,k)}, V^{(j<i,k)}, V^{(j\geq i,k-1)}, \mathcal{M}^{k-1}\right), \tag{3.1}$$

which is the classical LASSO problem with the variable $\mathcal{H}^{(i,k)}$. Although (3.1) is convex, there is no closed-form solution. To solve problem (3.1), we linearize the quadratic term of its objective function with an extrapolation point $\hat{\mathcal{H}}^{(i,k)}$ as follows

$$\ell\left(\mathcal{H}^{(i,k)}, V^{(j<i,k)}, V^{(j\geq i,k-1)}, \mathcal{M}^{k-1}\right) \approx \ell\left(\hat{\mathcal{H}}^{(i,k)}, V^{(j<i,k)}, V^{(j\geq i,k-1)}, \mathcal{M}^{k-1}\right) + \left\langle \nabla_{\mathcal{H}}\ell, \mathcal{H}^{(i,k)} - \hat{\mathcal{H}}^{(i,k)}\right\rangle + \frac{L_{\mathcal{H}}^{(i,k)}}{2}\left\|\mathcal{H}^{(i,k)} - \hat{\mathcal{H}}^{(i,k)}\right\|_F^2. \tag{3.2}$$

We update core tensor $\mathcal{H}^{(i,k)}$ by

$$\begin{aligned}&\underset{\mathcal{H}^{(i,k)}}{\operatorname{argmin}}\ \mu\lambda \left\|\mathcal{H}^{(i,k)}\right\|_{l_1} + \left\langle \nabla_{\mathcal{H}}\ell, \mathcal{H}^{(i,k)} - \hat{\mathcal{H}}^{(i,k)} \right\rangle + \frac{L_{\mathcal{H}}^{(i,k)}}{2} \left\|\mathcal{H}^{(i,k)} - \hat{\mathcal{H}}^{(i,k)}\right\|_F^2 \\ =&\underset{\mathcal{H}^{(i,k)}}{\operatorname{argmin}}\ \mu\lambda \left\|\mathcal{H}^{(i,k)}\right\|_{l_1} + \frac{L_{\mathcal{H}}^{(i,k)}}{2} \left\|\mathcal{H}^{(i,k)} - \frac{L_{\mathcal{H}}^{(i,k)}\hat{\mathcal{H}}^{(i,k)} - \nabla_{\mathcal{H}}\ell}{L_{\mathcal{H}}^{(i,k)}}\right\|_F^2 \\ =&T_{\frac{\mu\lambda}{L_{\mathcal{H}}^{(i,k)}}}\left(\frac{L_{\mathcal{H}}^{(i,k)}\hat{\mathcal{H}}^{(i,k)} - \nabla_{\mathcal{H}}\ell}{L_{\mathcal{H}}^{(i,k)}}\right).\end{aligned} \tag{3.3}$$

Here, we take $\hat{\mathcal{H}}^{(i,k)} = \mathcal{H}^{(i,k)} + \omega_{\mathcal{H}}^{(i,k)}\left(\mathcal{H}^{(i-1,k)} - \mathcal{H}^{(i-2,k)}\right)$ and

$$L_{\mathcal{H}}^{(i,k)} = \left\|V^{(1,k)}\right\|^2 \times \cdots \times \left\|V^{(i-1,k)}\right\|^2 \times \left\|V^{(i,k-1)}\right\|^2 \times \cdots \times \left\|V^{(m,k-1)}\right\|^2 + 1.$$

Update $W^{(i,k)}$

Matrix $W^{(i,k)}$ is updated by

$$
\begin{aligned}
W^{(i,k)} &:= \arg\min_{W^{(i,k)}} \ \mu\lambda \left\| W^{(i,k)} \right\|_{l_1} + \frac{1}{2} \left\| W^{(i,k)} - V^{(i,k-1)} \right\|_F^2 \\
&= T_{\mu\lambda}\left(V^{(i,k-1)} \right).
\end{aligned}
\tag{3.4}
$$

Update $V^{(i,k)}$

Factor matrix $V^{(i,k)}$ is updated by optimizing

$$\min_{V^{(i,k)}} \mu \left\| V^{(i,k)} \right\|_{*,r_i} + \ell\left(\mathcal{H}^{(i,k)}, V^{(j<i,k)}, V^{(j\geq i,k-1)}, \mathcal{M}^{k-1}\right) + \frac{1}{2}\left\| V^{(i,k)} - W^{(i,k)} \right\|_F^2 + \frac{\xi}{2}\left\| V^{(i,k)} - V^{(i,k-1)} \right\|_F^2, \tag{3.5}$$

where ξ is a positive constant such that

$$\mu \left\| V^{(i,k)} \right\|_{*,r_i} + \frac{\xi}{2}\left\| V^{(i,k)} - V^{(i,k-1)} \right\|_F^2$$

is convex.

Let

$$v_s^{(i,k)} = \begin{cases} 0 & s = 1, \ldots, r_i, \\ \mu & s = r_i + 1, \ldots, n_i. \end{cases}$$

Then problem (3.5) can be written as

$$\begin{aligned} \min_{V^{(i,k)}} \ & \sum_{s=1}^{n_i} v_s^{(i,k)} \sigma_s \left(V^{(i,k)} \right) + \frac{1}{2} \left\| V^{(i,k)} B_i^k - M_{(i)}^{k-1} \right\|_F^2 \\ & + \frac{1}{2} \left\| V^{(i,k)} - W^{(i,k)} \right\|_F^2 + \frac{\xi}{2} \left\| V^{(i,k)} - V^{(i,k-1)} \right\|_F^2, \end{aligned} \tag{3.6}$$

where

$$B_i^k = \mathcal{H}^{(i,k)} \left(V^{(m,k-1)} \otimes \cdots \otimes V^{(i+1,k-1)} \otimes V^{(i-1,k)} \otimes \cdots \otimes V^{(1,k)} \right)^T.$$

To get a closed-form approximated solution of $V^{(i,k)}$, we linearize the second term of (3.6) as follows

$$\begin{aligned} &\frac{1}{2}\left\| V^{(i,k)} B_i^k - M_{(i)}^{k-1} \right\|_F^2 \\ &\approx \frac{1}{2}\left\| \hat{V}^{(i,k)} B_i^k - M_{(i)}^{k-1} \right\|_F^2 + \left\langle \nabla_{V^{(i)}} \ell, V^{(i,k)} - \hat{V}^{(i,k)} \right\rangle + \frac{L_i^k}{2} \left\| V^{(i,k)} - \hat{V}^{(i,k)} \right\|_F^2, \end{aligned} \tag{3.7}$$

where

$$\hat{V}^{(i,k)} = V^{(i,k-1)} + \omega_i^k \left(V^{(i,k-1)} - V^{(i,k-2)} \right), \ L_i^k = \left\| B_i^k \left(B_i^k \right)^T \right\|_2 + 1$$

Plugging (3.7) into (3.6), we update factor matrix $V^{(i,k)}$ by

$$\begin{aligned}
&\underset{V^{(i,k)}}{\operatorname{argmin}} \sum_{s=1}^{n_i} v_s^{(i,k)} \sigma_s\left(V^{(i,k)}\right) + \left\langle \nabla_{V^{(i)}} \ell, V^{(i,k)} - \hat{V}^{(i,k)} \right\rangle \\
&+ \frac{L_i^k}{2} \left\| V^{(i,k)} - \hat{V}^{(i,k)} \right\|_F^2 + \frac{1}{2} \left\| V^{(i,k)} - W^{(i,k)} \right\|_F^2 + \frac{\xi}{2} \left\| V^{(i,k)} - V^{(i,k-1)} \right\|_F^2 \\
=&\underset{V^{(i,k)}}{\operatorname{argmin}} \sum_{s=1}^{n_i} v_s^{(i,k)} \sigma_s\left(V^{(i,k)}\right) \\
&+ \frac{L_i^k + \xi + 1}{2} \left\| V^{(i,k)} - \frac{L_i^k \hat{V}^{(i,k)} - \nabla_{V^{(i)}} \ell + W^{(i,k)} + \xi V^{(i,k-1)}}{L_i^k + \xi + 1} \right\|_F^2 \\
=&S_{\frac{1}{L_i^k + \xi + 1}} \left(\frac{L_i^k \hat{V}^{(i,k)} - \nabla_{V^{(i)}} \ell + W^{(i,k)} + \xi V^{(i,k-1)}}{L_i^k + \xi + 1}, \boldsymbol{v}^{(i,k)} \right).
\end{aligned} \tag{3.8}$$

Update $\mathcal{M}^k$

Tensor $\mathcal{M}^k$ is updated by

$$\begin{aligned}\mathcal{M}^k &= \underset{P_\Omega(\mathcal{M}-\overline{\mathcal{X}})=0}{\arg\min} \frac{1}{2\mu} \left\| \mathcal{H}^k \times_1 V^{(1,k)} \times_2 \cdots \times_m V^{(m,k)} - \mathcal{M} \right\|_F^2 \\ &= P_\Omega(\overline{\mathcal{X}}) + P_{\Omega^c}\left(\mathcal{H}^k \times_1 V^{(1,k)} \times_2 \cdots \times_m V^{(m,k)} \right). \end{aligned} \tag{3.9}$$

Algorithm

Algorithm 3.1 Low-Tucker-Rank Tensor Completion (LTRTC)

Input: The tensor data $\overline{\mathcal{X}}$, the observed set Ω, rank $r_i, i \in [m]$ and parameters μ.
Initialize: $\left(\mathcal{H}^{-1}, V^{(1,-1)}, \ldots, V^{(m,-1)}\right) = \left(\mathcal{H}^{0}, V^{(1,0)}, \ldots, V^{(m,0)}\right)$.
While not converge do

Let $\mathcal{H}^{(-1,1)} = \mathcal{H}^{(0,1)} = \mathcal{H}^{0}$, $\mathcal{H}^{(-1,k)} = \mathcal{H}^{(m-1,k-1)}$, $\mathcal{H}^{(0,k)} = \mathcal{H}^{(m,k-1)}$ $(k \geq 2)$.

For $i = 1, \ldots, m$ **do**

- **Step 1.** Compute $L_{\mathcal{H}}^{(i,k)}$ and set $\omega_{\mathcal{H}}^{(i,k)}$.
- **Step 2.** Let $\hat{\mathcal{H}}^{(i,k)} = \mathcal{H}^{(i-1,k)} + \omega_{\mathcal{H}}^{(i,k)} \left(\mathcal{H}^{(i-1,k)} - \mathcal{H}^{(i-2,k)}\right)$.
- **Step 3.** Update $\mathcal{H}^{(i,k)}$ according to (3.3).
- **Step 4.** Update $W^{(i,k)}$ according to (3.4).
- **Step 5.** Compute L_i^k and set ω_i^k.
- **Step 6.** Let $\hat{V}^{(i,k)} = V^{(i,k-1)} + \omega_i^k \left(V^{(i,k-1)} - V^{(i,k-2)}\right)$.
- **Step 7.** Update $V^{(i,k)}$ according to (3.8).

end

Let $\mathcal{H}^{k} = \mathcal{H}^{(m,k)}$.

Update $\mathcal{M}^k$ according to (3.9).

$k \leftarrow k + 1$.

end while
Output: $\left(\mathcal{H}^{k}, V^{(1,k)}, \ldots, V^{(m,k)}, W^{(1,k)}, \ldots, W^{(m,k)}, \mathcal{M}^{k}\right)$.

The convergence of LTRTC:

Theorem

Suppose that the sequence $\{\mathcal{H}^k, V^{(1,k)}, \ldots, V^{(m,k)}, W^{(1,k)}, \ldots, W^{(m,k)}, \mathcal{M}^k\}$ *generated by Algorithm LTRTC is bounded. Then any of its accumulation point is a stationary point of problem (2.5).*

Improvement with temporal characteristics

In the real world, some characteristics are included in the related tensor data. Hence problem (2.5) can be improved as follows

$$\begin{array}{ll} \min & \sum_{i=1}^{m} \left(\mu \| V^{(i)} \|_{*,r_i} + \mu\lambda \| V^{(i)} \|_1 + \frac{\beta_i}{2} \left\| Q^{(i)} V^{(i)} \right\|_F^2 \right) \\ & + \mu\lambda \|\mathcal{H}\|_1 + \ell\left(\mathcal{H}, V^{(1)}, \ldots, V^{(m)}, \mathcal{M}\right) \\ \text{s.t.} & P_\Omega\left(\mathcal{M} - \overline{\mathcal{X}}\right) = 0. \end{array} \tag{3.10}$$

Here

$$Q^{(i)} = \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ -0.5 & 1 & -0.5 & \cdots & 0 & 0 \\ 0 & -0.5 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -0.5 \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{pmatrix}_{n_i \times n_i}.$$

Contents

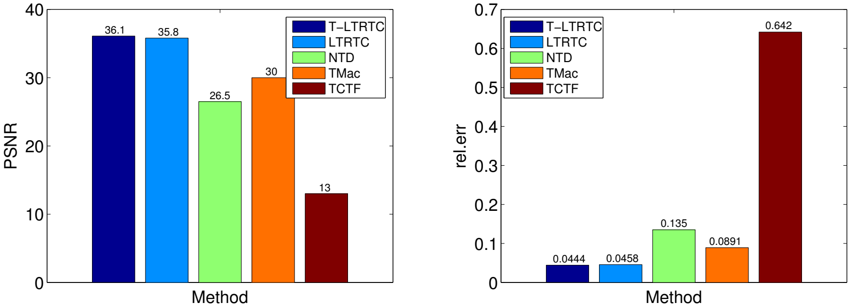
Image simulation(The Berkeley Segmentation Database)

Table: Numerical results for The Berkeley Segmentation Database.

Method	Airplane		Church		Woman		Children	
	PSNR	rel.err	PSNR	rel.err	PSNR	rel.err	PSNR	rel.err
T-LTRTC	**30.3**	8.75e-02	**35.5**	2.46e-02	**31.3**	7.28e-02	**36.9**	4.41e-02
LTRTC	28.0	1.14e-01	33.2	3.22e-02	29.4	9.05e-02	34.0	6.18e-02
NTD	25.3	1.56e-01	27.2	6.41e-02	25.6	1.41e-01	28.4	1.17e-01
TMac	25.7	1.48e-01	28.4	5.61e-02	27.7	1.10e-01	25.0	1.72e-01
TCTF	20.6	2.68e-01	22.4	1.11e-01	17.0	3.78e-01	19.8	3.16e-01

Figure: Completion results of The Berkeley Segmentation Database.

Image simulation(MRI Volume Dataset)



(a) PSNR *(b)* rel.err

Figure: Histogram of representation results for the MRI Volume Dataset.

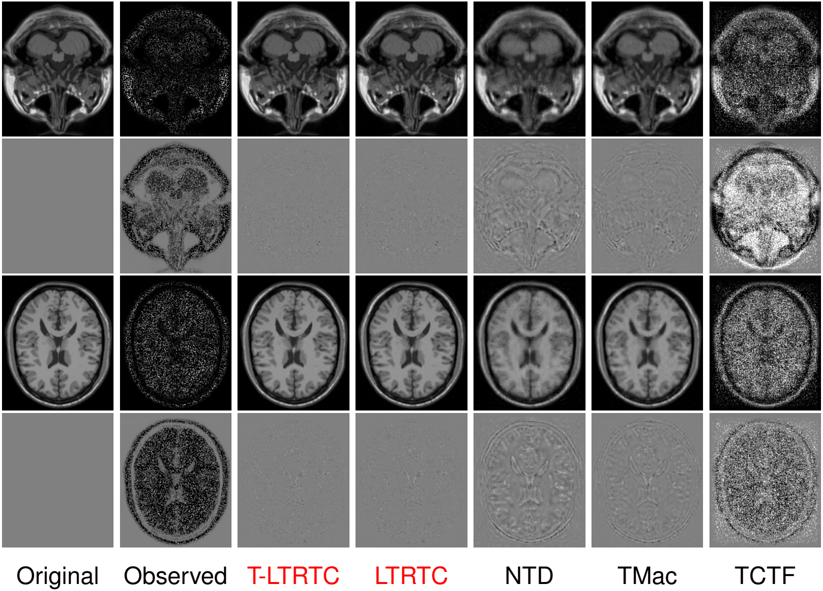


Figure: Completion results of The MRI Volume Dataset.

Image simulation(California Institute of Technology Color Face Image Library)

Table: Numerical results for face pictures.

Image	T-LTRTC		LTRTC		NTD		TMac		TCTF	
	PSNR	rel.err	PSNR	rel.err	PSNR	rel.err	PSNR	rel.err	PSNR	rel.err
Face1	**32.4**	3.55e-02	29.9	4.72e-02	23.7	9.60e-02	25.9	7.47e-02	23.7	9.66e-02
Face2	**32.4**	3.53e-02	29.8	4.77e-02	24.2	9.01e-02	25.6	7.70e-02	25.1	8.21e-02
Face3	**30.5**	4.49e-02	28.2	5.90e-02	22.9	1.08e-01	24.6	8.89e-02	22.5	1.13e-01
Face4	**31.8**	5.11e-02	30.4	6.01e-02	26.3	9.60e-02	26.0	9.92e-02	25.3	1.08e-01
Face5	**30.8**	5.85e-02	29.5	6.79e-02	25.9	1.03e-01	25.5	1.08e-01	24.9	1.15e-01
Face6	**33.9**	2.77e-02	32.4	3.33e-02	23.5	9.26e-02	27.3	5.98e-02	28.6	5.11e-02

Video simulation

Table: Numerical results for video inpainting.

Video	T-LTRTC		LTRTC		NTD		TMac	
	PSNR	rel.err	PSNR	rel.err	PSNR	rel.err	PSNR	rel.err
Suzie	**35.1**	3.86e-02	34.3	4.26e-02	28.6	8.22e-02	29.3	7.53e-02
News	**35.0**	4.76e-02	34.6	5.00e-02	26.3	1.30e-01	28.3	1.03e-01
Carphone	**32.9**	4.80e-02	32.5	5.06e-02	27.4	9.03e-02	28.9	7.60e-02

Figure: Uniformly sampled video inpainting.

In real life, there may be a basic lack of data in a certain frame of the video, and worse, the basic lack of data in several consecutive frames. In order to check the recovery effect of our model on this situation, we lose the data of the 8th frame, the 8-9th frame, the 7-9th frame, the 6th-10th frame, and we uniformly select 0.1% of the lost data the samples and other data points without missing frames are known.

Table: Numerical results for the masked video inpainting.

Lost frame	T-LTRTC		LTRTC		NTD		TMac	
	PSNR	rel.err	PSNR	rel.err	PSNR	rel.err	PSNR	rel.err
8	**59.8**	2.27e-03	33.9	4.47e-02	30.1	6.93e-02	26.5	1.04e-01
8-9	**55.1**	3.88e-03	30.9	6.27e-02	26.6	1.03e-01	23.7	1.45e-01
7-9	**50.4**	6.64e-03	27.3	9.56e-02	24.3	1.35e-01	21.0	1.97e-01
6-10	**35.9**	3.55e-02	26.0	1.10e-02	23.3	1.52e-01	19.4	2.37e-01

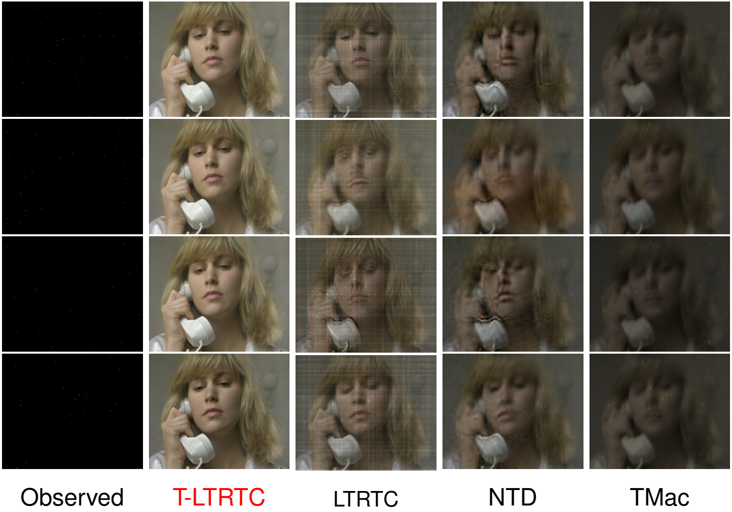


Figure: Recovered video of the masked video. From top to bottom, the data of the 8th frame, the 8th-9th frame, the 7th-9th frame, and the 6th-10th frame are lost.

Thank you!